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Acoustic phonon quantization in buried waveguides and resonators

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Abstract. Starting from a classical Hamiltonian for nonhomogeneous elastic media, a procedure is developed for acoustic phonon quantization in resonators as well as linear and planar waveguides. The formalism is illustrated in an example of acoustic phonon modes in a buried cylindrical waveguide. The deformation potential Hamiltonian for electron–acoustic phonon interaction is also obtained.

1. Introduction

During the last decade much effort has been devoted to understanding of influence of spatial quantization on the vibrational properties of semiconductor heterostructures and superlattices. While optical phonon confinement has been analysed in great detail [1], it is only recently that much attention has been focused on the more subtle effects of acoustic phonon quantization in restricted geometries.

Early works on acoustic phonon properties in superlattices were devoted mainly to the study of acoustic mode folding [2]. More recently, Tamura and co-workers investigated the resonant transmission of acoustic wavepackets in superlattices and double-barrier systems [3]. Kochelap and Gülseren [4] have modelled the localization of acoustical modes due to electron–phonon interactions within a two-dimensional electron gas. Also, in a number of works the modification of acoustic modes in heterostructures has been studied [5–7] within the formalism of *classical* elasticity theory [8, 9].

Recent advances in material growth techniques have resulted in the fabrication of free-standing nanostructures [11] and have provided the possibility of observation of acoustic phonon confinement effects. Wybourne and co-workers [12] have presented experimental evidence of acoustic phonon confinement effects in self-supported thin films. Subsequently, these experimental findings motivated theoretical investigations on the role of acoustic phonon *quantization* in free-standing nanostructures [13–15]. Furthermore, acoustic phonon modes had been properly quantized in self-supported whiskers [13, 14], dots [13] and slabs [15], and electron–phonon interactions in such systems have been studied.

To the best of our knowledge, the quantum mechanical treatment of acoustic phonons has been provided only for free-boundary, homogeneous waveguides and resonators. On the other hand, semiconductor quantum wells, wires and dots are conventionally grown embedded in another material. It has been also proposed [7] that a buried quantum wire could serve as an acoustic fibre in semiconductor acoustoelectronic devices. Thus, proper quantization of acoustic phonon modes in buried structures is essential for an accurate treatment of mesoscopic and coherent phenomena in low-dimensional systems.

In this paper we develop a quantization procedure for acoustic vibrations confined in linear or planar waveguides as well as resonators. Expressions for the displacement operator are derived starting from the most general form of the Hamiltonian for an inhomogeneous elastic medium. The quantization formalism is illustrated by deriving the acoustic phonon spectrum for a buried cylindrical wire. We also present the resulting deformation potential Hamiltonian responsible for the electron–phonon interaction.

This paper is organized as follows. In section 2 we obtain general rules for acoustic phonon quantization in resonators and waveguides. Section 3 deals with application of the quantization procedure to a buried cylindrical fibre. Finally, we summarize the results obtained in section 4.

2. Quantization procedure

In section 2.1 we present the procedure for acoustic phonon quantization in resonators; section 2.2 contains the rule for quantization of phonons confined in one or two dimensions (acoustic waveguide), and section 2.3 presents the deformation potential Hamiltonian for acoustic phonons.

2.1. Acoustic phonon quantization in resonators

We consider the quantization of acoustic modes localized in a certain region of elastic material (resonator). The most general form of the Hamiltonian for an inhomogeneous elastic medium is given by [9, 10]

$$\mathcal{H} = \frac{1}{2}\int \mathrm{d}^3 R\, \left[\rho(\boldsymbol{R})\dot{u}_i\dot{u}_i + \lambda_{ijkl}(\boldsymbol{R})\,\frac{\partial u_i}{\partial x_j}\,\frac{\partial u_k}{\partial x_l}\right] \tag{1}$$

where u_i are components of the displacement vector $\boldsymbol{u}(\boldsymbol{R}, t)$; also $\rho(\boldsymbol{R})$ is a mass density, and $\lambda_{ijkl}(\boldsymbol{R})$ is the elastic stiffness tensor of the medium.

The corresponding equations of motion take the following form:

$$\rho(\boldsymbol{R})\ddot{u}_i(\boldsymbol{R}, t) = \frac{\partial}{\partial x_j}\left[\lambda_{ijkl}(\boldsymbol{R})\,\frac{\partial u_k(\boldsymbol{R}, t)}{\partial x_l}\right]. \tag{2}$$

The general solution of equation (2) in the case of localized vibrations can be presented as a linear combination of normal modes $\boldsymbol{w}_n(\boldsymbol{R})$ which are labelled by a discrete number n. For the quantization of the elastic vibrations it is convenient to deal with a real (rather than complex) displacement vector $\boldsymbol{u}(\boldsymbol{R}, t)$; thus,

$$\boldsymbol{u}(\boldsymbol{R}, t) = \sum_n \left[c_n \boldsymbol{w}_n(\boldsymbol{R})\,\mathrm{e}^{-\mathrm{i}\omega_n t} + c_n^* \boldsymbol{w}_n^*(\boldsymbol{R})\,\mathrm{e}^{\mathrm{i}\omega_n t}\right]. \tag{3}$$

In order to define unambiguously the coefficients c_n and c_n^* in equation (3), the normalization rule for the modes $\boldsymbol{w}_n(\boldsymbol{R}, t)$ should be specified. Since the requirement of constant energy for confined vibrations implies their mutual orthogonality with the density $\rho(\boldsymbol{R})$ as a weight factor [10], the following orthonormality conditions can be imposed:

$$\int \mathrm{d}^3 R\ \rho(\boldsymbol{R})\boldsymbol{w}_n^*(\boldsymbol{R}) \cdot \boldsymbol{w}_{n'}(\boldsymbol{R}) = \delta_{n,n'}. \tag{4}$$

In order to obtain the quantization rules the Hamiltonian (1) must be expressed in terms of the amplitudes of the modes, c_n and c_n^*. Integrating by parts the second term in equation (1), we find, with the help of equations (2)–(4), the following representation of the Hamiltonian:

$$\mathcal{H} = \sum_n \omega_n (c_n c_n^* + c_n^* c_n). \tag{5}$$

The classical form (5) for elastic vibrations corresponds to the free-phonon Hamiltonian $\hat{\mathcal{H}}$ in the second-quantization representation

$$\hat{\mathcal{H}} = \sum_n \hbar\omega_n \left[\hat{b}_n(t)\hat{b}_n^\dagger(t) + \frac{1}{2}\right] = \sum_n \frac{\hbar\omega_n}{2}\left[\hat{b}_n(t)\hat{b}_n^\dagger(t) + \hat{b}_n^\dagger(t)\hat{b}_n(t)\right] \tag{6}$$

where the time-dependent annihilation and creation operators satisfy standard commutation relations:

$$\hat{b}_n(t)\hat{b}_{n'}^\dagger(t) - \hat{b}_{n'}^\dagger(t)\hat{b}_n(t) = \delta_{n,n'}.$$

In essence, the quantization procedure amounts to (see, e.g., [16]) comparison of the classical Hamiltonian of equation (5) with the quantum mechanical Hamiltonian of equation (6). This comparison leads to the correspondence rule

$$c_n\, \mathrm{e}^{-\mathrm{i}\omega_n t} \rightarrow \sqrt{\frac{\hbar}{2\omega_n}}\hat{b}_n(t) \qquad c_n^*\, \mathrm{e}^{\mathrm{i}\omega_n t} \rightarrow \sqrt{\frac{\hbar}{2\omega_n}}\hat{b}_n^\dagger(t). \tag{7}$$

Finally, applying the rule (7) to the classical displacement vector in *resonators* given by equation (3), we find, in second-quantization representation,

$$\hat{\boldsymbol{u}}(\boldsymbol{R}, t) = \sum_n \sqrt{\frac{\hbar}{2\omega_n}}\left[\boldsymbol{w}_n(\boldsymbol{R})\,\hat{b}_n + \boldsymbol{w}_n^*(\boldsymbol{R})\,\hat{b}_n^\dagger\right]. \tag{8}$$

For simplicity we have suppressed the time dependence of the operators $\hat{b}(t)$ and $\hat{b}^\dagger(t)$. The phonon wavefunctions $\boldsymbol{w}_n(\boldsymbol{R})$ are normalized according to equation (4).

2.2. Phonon quantization in acoustic waveguides

The quantization rules for acoustic waveguides are easily obtained from comparison of equations (8), (4) for phonons confined in all dimensions with corresponding expressions, describing the homogeneous case [16]. Introducing the notation $\boldsymbol{R} = (\boldsymbol{r}, z)$ and $\boldsymbol{Q} = (\boldsymbol{q}, q_z)$, we consider (i) planar waveguides homogeneous in the plane with constant z and (ii) linear waveguides which are homogeneous along the z-direction.

Thus, for *linear* acoustic waveguides, the operator for the displacement vector is given by

$$\hat{\boldsymbol{u}}(\boldsymbol{R}, t) = \sum_{n,q_z} \sqrt{\frac{\hbar}{2\omega_{nq_z}}}\left[\boldsymbol{w}_{n,q_z}(\boldsymbol{r})\,\hat{b}_{n,q_z} + \boldsymbol{w}_{n,-q_z}^*(\boldsymbol{r})\,\hat{b}_{n,-q_z}^\dagger\right]\frac{\mathrm{e}^{\mathrm{i}q_z z}}{\sqrt{\mathcal{L}}}. \tag{9}$$

Here $\mathcal{L}$ is a normalization length; the phonon frequency ω_{nq_z} and eigenvectors $\boldsymbol{w}_{n,q_z}(\boldsymbol{r})$ should be found by solving of the equations of motion (2) with proper boundary condition. The normalization condition for the eigenvectors is

$$\int \mathrm{d}^2 r\ \rho(\boldsymbol{r})\boldsymbol{w}_{n,q_z}^*(\boldsymbol{r}) \cdot \boldsymbol{w}_{n',q_z}(\boldsymbol{r}) = \delta_{n,n'}. \tag{10}$$

In the same fashion, for a *planar* waveguide the displacement operator is equal to

$$\hat{\boldsymbol{u}}(\boldsymbol{R}, t) = \sum_{n,\boldsymbol{q}} \sqrt{\frac{\hbar}{2\omega_{nq}}}\left[\boldsymbol{w}_{n,\boldsymbol{q}}(z)\hat{b}_{n,\boldsymbol{q}} + \boldsymbol{w}_{n,-\boldsymbol{q}}^*(z)\hat{b}_{n,-\boldsymbol{q}}^\dagger\right]\frac{\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}}{\sqrt{\mathcal{S}}} \tag{11}$$

where $\mathcal{S}$ is a normalization area and the eigenvectors $\boldsymbol{w}_{n,\boldsymbol{q}}(z)$ must be normalized according to the prescription

$$\int \mathrm{d}z\ \rho(z)\boldsymbol{w}^*_{n,\boldsymbol{q}}(z)\cdot\boldsymbol{w}_{n',\boldsymbol{q}}(z) = \delta_{n,n'}. \tag{12}$$

2.3. The deformation potential

Interaction via the deformation potential is usually a dominant mechanism for electron–acoustic phonon scattering in crystals. The deformation potential Hamiltonian can be written in a general form as

$$\hat{\mathcal{H}}_{\mathrm{def}} = \Xi_{\mathrm{ac}}\ \mathrm{div}\ \hat{\boldsymbol{u}}(\boldsymbol{R}, t) \tag{13}$$

where Ξ_{ac} is an acoustic deformation potential constant, and an implicit form of the displacement operator $\hat{\boldsymbol{u}}$ is given by equations (8), (9), or (11).

3. The cylindrical waveguide

To illustrate the application of the acoustic phonon quantization procedure described in the previous section, we consider the example of a linear cylindrical waveguide in an isotropic medium. Section 3.1 provides the general solution for quantized acoustic modes in this system; in section 3.2, we investigate the particular case of axisymmetric modes in more detail. Section 3.3 contains the expression for the quantized deformational potential Hamiltonian.

3.1. Basic equations

We consider a buried cylindrical waveguide of radius a occupying the region $r < a$. The inner (outer) region of the waveguide is filled with an isotropic medium characterized by constant mass density ρ_1 (ρ_2) and Lamé coefficients λ_1, μ_1 (λ_2, μ_2) which specify the elastic stiffness tensor of each isotropic medium in equation (1) according to $\lambda_{ijkl} = \lambda\,\delta_{ij}\delta_{kl} + 2\mu\,\delta_{ik}\delta_{jl}$.

The general solution of the classical equations of motion (2) in each region can be written [9] in terms of three scalar potentials ϕ, ψ and χ:

$$\boldsymbol{u} = \boldsymbol{\nabla}\phi + \boldsymbol{\nabla}\times(\boldsymbol{e}_z\psi) + a\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times(\boldsymbol{e}_z\chi). \tag{14}$$

Here $\boldsymbol{e}_z$ is a unit vector along the z-axis. Each potential ϕ (ψ, χ) satisfies a scalar wave equation with propagation speed equal to the longitudinal (transverse) sound speed s_l (s_t) given by

$$s_{l\nu} = \sqrt{(\lambda_\nu + 2\mu_\nu)/\rho_\nu} \qquad s_{t\nu} = \sqrt{\mu_\nu/\rho_\nu}.$$

In these expressions, $\nu = 1$ ($\nu = 2$) corresponds to the material constant of the inner (outer) region.

We seek the solutions of equation (2) as harmonic vibrations with frequency ω, wavevector $q_z \equiv q/a$, and azimuthal number m, confined in the vicinity of the waveguide. Using the cylindrical coordinate system, $\boldsymbol{R} = (r, \varphi, z)$, we take the scalar potentials in the inner region ($r < a$) as

$$\begin{bmatrix}\phi\\ \psi\\ \chi\end{bmatrix} = \frac{1}{a}\begin{bmatrix}\mathrm{i}c_{l1}J_m(k_l r/a)\\ C_{t1}J_m(k_t r/a)\\ c_{t1}J_m(k_t r/a)\end{bmatrix}\mathrm{e}^{\mathrm{i}m\varphi+\mathrm{i}qz/a-\mathrm{i}\omega t} \tag{15}$$

and for the outer region ($r > a$)

$$\begin{bmatrix} \phi \\ \psi \\ \chi \end{bmatrix} = \frac{1}{a} \begin{bmatrix} \mathrm{i}c_{l2} K_m(\kappa_l r/a) \\ C_{t2} K_m(\kappa_t r/a) \\ c_{t2} K_m(\kappa_t r/a) \end{bmatrix} \mathrm{e}^{\mathrm{i}m\varphi + \mathrm{i}qz/a - \mathrm{i}\omega t}. \tag{16}$$

Here the inverse wavelengths $k_{l,t}$ and localization length $\kappa_{l,t}$ in the radial direction are defined as

$$k_{l,t}^2 = q^2 - \omega^2 a^2/s_{(l,t)1}^2 \qquad \kappa_{l,t}^2 = \omega^2 a^2/s_{(l,t)2}^2 - q^2.$$

For definiteness, equations (15) and (16) are written under the assumption that $k_{l,t}^2, \kappa_{l,t}^2 > 0$ which corresponds to the case of confined acoustic vibrations. Other possible cases are treated formally in the same fashion using analytical properties of Bessel's functions, and are discussed in the appendix.

Substituting equations (15) and (16) into equation (14), we find the implicit form of the displacement vector:

$$\boldsymbol{u}(r, \varphi, z, t) = \boldsymbol{u}(r)\mathrm{e}^{\mathrm{i}m\varphi + \mathrm{i}qz/a - \mathrm{i}\omega t} \tag{17}$$

where inside the waveguide ($r < a$)

$$\begin{aligned} -\mathrm{i}u_r(r) &= c_{l1} k_l J_m'(k_l r/a) + C_{t1} m \frac{a}{r} J_m(k_t r/a) + c_{t1} q k_t J_m'(k_t r/a) \\ -u_\varphi(r) &= c_{l1} m \frac{a}{r} J_m(k_l r/a) + C_{t1} k_t J_m'(k_t r/a) + c_{t1} m q \frac{a}{r} J_m(k_t r/a) \\ -u_z(r) &= c_{l1} q J_m(k_l r/a) - c_{t1} k_t^2 J_m(k_t r/a) \end{aligned} \tag{18}$$

while for $r > a$ we have

$$\begin{aligned} -\mathrm{i}u_r(r) &= c_{l2} \kappa_l K_m'(\kappa_l r/a) + C_{t2} m \frac{a}{r} K_m(\kappa_t r/a) + c_{t2} q \kappa_t K_m'(\kappa_t r/a) \\ -u_\varphi(r) &= c_{l2} m \frac{a}{r} K_m(\kappa_l r/a) + C_{t2} \kappa_t K_m'(\kappa_t r/a) + c_{t2} m q \frac{a}{r} K_m(\kappa_t r/a) \\ -u_z(r) &= c_{l2} q K_m(\kappa_l r/a) + c_{t2} \kappa_t^2 K_m(\kappa_t r/a). \end{aligned} \tag{19}$$

Applying standard boundary conditions (continuity of the displacement and normal components of the stress tensor at the boundary $r = a$) gives the following 6×6 characteristic equation for the acoustic vibrations of a buried cylindrical fibre:

$$\begin{bmatrix} \mathsf{U}_1 & -\mathsf{U}_2 \\ \mu_1 \mathsf{F}_1 & -\mu_2 \mathsf{F}_2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = 0 \tag{20}$$

where $\boldsymbol{C}_\nu = [c_{l\nu}, C_{t\nu}, c_{t\nu}]^{\mathrm{T}}$; displacement matrices at $r = a$ are given by

$$\mathsf{U}_\nu = \begin{bmatrix} L_\nu & m t_\nu & q T_\nu \\ -m l_\nu & -T_\nu & -m q t_\nu \\ -q l_\nu & 0 & k_{\nu t}^2 t_\nu \end{bmatrix} \tag{21}$$

and matrices F_ν, related to the elastic forces at the interface, are equal to

$$\mathsf{F} = \begin{bmatrix} -2qL_\nu & -mqt_\nu & (k_{\nu t}^2 - q^2) T_\nu \\ 2m(l_\nu - L_\nu) & (k_{\nu t}^2 - 2m^2) t_\nu + 2T_\nu & 2mq(t_\nu - T_\nu) \\ (2m^2 + q^2 - k_{\nu l}^2) l_\nu - 2L_\nu & 2m(T_\nu - t_\nu) & 2q[(m^2 - k_{\nu t}^2) t_\nu - T_\nu] \end{bmatrix}. \tag{22}$$

Here $k^2_{1(l,t)} = k^2_{l,t}$, $k^2_{2(l,t)} = -\kappa^2_{l,t}$, and

$$l_1 = J_m(k_l) \qquad L_1 = k_l J'_m(k_l) \qquad t_1 = J_m(k_t) \qquad T_1 = k_t J'_m(k_t)$$
$$l_2 = K_m(\kappa_l) \qquad L_2 = \kappa_l K'_m(\kappa_l) \qquad t_2 = K_m(\kappa_t) \qquad T_2 = \kappa_t K'_m(\kappa_t).$$

Equations (20)–(22) define the dispersion law and eigenmodes of elastic vibrations in a buried cylindrical waveguide for arbitrary azimuthal number m.

Finally, taking into account the notation introduced above, we can rewrite equation (9) for the displacement operator as

$$\hat{\boldsymbol{u}}(\boldsymbol{R}, t) = \sum_{mn,q} \sqrt{\frac{\hbar}{2\omega_{mn,q}\mathcal{L}}} \left[\boldsymbol{w}_{mn,q}(r)\hat{b}_{mn,q} + \boldsymbol{w}^*_{mn,-q}(r)\hat{b}^\dagger_{mn,-q} \right] \mathrm{e}^{\mathrm{i}m\varphi + \mathrm{i}qz/a}. \tag{23}$$

Here, the discrete quantum number n enumerates phonon modes with the same m and q, and the normal modes $\boldsymbol{w}$ can be represented conveniently in the following form:

$$\boldsymbol{w}_{mn,q}(\boldsymbol{r}) \equiv \boldsymbol{w}_{mn,q}(r)\mathrm{e}^{\mathrm{i}m\varphi} = \boldsymbol{u}(r)\mathrm{e}^{\mathrm{i}m\varphi}/\sqrt{\pi a^2 \mathcal{N}} \tag{24}$$

where the normalization constant $\mathcal{N}$ must be determined from the condition of equation (10).

Below we consider in detail the important case of axisymmetric ($m = 0$) vibrations in a cylindrical waveguide.

3.2. Axisymmetric vibrations

In case of axisymmetric vibrations, $m = 0$, the 6×6 determinant corresponding to equation (20) decouples into 2×2 and 6×6 blocks, specifying axisymmetric *torsional* and *radial–axial* modes. Below we present the expressions for these two types of axisymmetric normal modes $\boldsymbol{w}_q(r)$, which appear in equation (9) for the displacement operator of a cylindrical waveguide.

3.2.1. Torsional modes. According to equations (18)–(22), the dispersion relation for the torsional vibrations is specified by the following transcendental equation:

$$\mu_1 k_t J_2(k_t)/J_1(k_t) = \mu_2 \kappa_t K_2(\kappa_t)/K_1(\kappa_t) \tag{25}$$

while the envelope function is given by $w_r = w_z = 0$ and

$$w_\varphi = \frac{1}{\sqrt{\pi a^2 \mathcal{N}_\varphi}} \begin{cases} K_1(\kappa_t) J_1(k_t r/a) & r < a \\ J_1(k_t) K_1(\kappa_t r/a) & r > a. \end{cases} \tag{26}$$

Here $\mathcal{N}_\varphi$ is the dimensionless normalization constant. Using the notation that $J_{m(l,t)} \equiv J_m(k_{l,t})$ and $K_{m(l,t)} \equiv K_m(\kappa_{l,t})$, we find from the normalization condition (10)

$$\mathcal{N}_\varphi = \rho_1 K^2_{1t}(J^2_{1t} - J_{0t}J_{2t}) + \rho_2 J^2_{1t}(K_{0t}K_{2t} - K^2_{1t}). \tag{27}$$

3.2.2. Radial–axial modes. For radial–axial axisymmetric vibrations the normal modes are given by $w_\varphi = 0$ and

$$\mathrm{i}w_r = \frac{1}{\sqrt{\pi a^2 \mathcal{N}_{rz}}} \begin{cases} c_{l1} k_l J_1(k_l r/a) + c_{t1} q k_t J_1(k_t r/a) & r < a \\ c_{l2} \kappa_l K_1(\kappa_l r/a) + c_{t2} q \kappa_t K_1(\kappa_t r/a) & r > a \end{cases} \tag{28}$$

$$-w_z = \frac{1}{\sqrt{\pi a^2 \mathcal{N}_{rz}}} \begin{cases} c_{l1} q J_0(k_l r/a) - c_{t1} k_t^2 J_0(k_t r/a) & r < a \\ c_{l2} q K_0(\kappa_l r/a) + c_{t2} \kappa_t^2 K_0(\kappa_t r/a) & r > a. \end{cases} \tag{29}$$

The dispersion relation and the relationship between coefficients $c_{\nu l}$, $c_{\nu t}$ are specified by the following eigenequation (cf. equations (20)–(22)):

$$\begin{bmatrix} -k_l J_{1l} & -qk_t J_{1t} & \kappa_l K_{1l} & q\kappa_t K_{1t} \\ -qJ_{0l} & k_t^2 J_{0t} & qK_{0l} & \kappa_t^2 K_{0t} \\ 2\mu_1 qk_l J_{1l} & \mu_2 k_t(q^2-k_t^2)J_{1t} & -2\mu_2 q\kappa_l K_{1l} & -\mu_2\kappa_t(\kappa_t^2+q^2)K_{1t} \\ \mu_1[AJ_{0l}+2k_lJ_{1l}] & 2\mu_1 qk_t[J_{1t}-k_tJ_{0t}] & -\mu_2[BK_{0l}+2\kappa_lK_{1l}] & -2\mu_2 q\kappa_t[J_{1t}+\kappa_tJ_{0t}] \end{bmatrix} \times \begin{bmatrix} c_{1l} \\ c_{1t} \\ c_{2l} \\ c_{2t} \end{bmatrix} = 0. \tag{30}$$

where $A = q^2 - k_l^2$ and $B = q^2 + \kappa_l^2$.

The normalization constant $\mathcal{N}_{rz}$ in equations (28) and (29) may be determined from the condition (10). After some calculations, we find

$$\begin{aligned}\mathcal{N}_{rz} = \rho_1\{&c_{1l}^2[q^2(J_{0l}^2 + J_{1l}^2) + k_l^2(J_{1l}^2 - J_{0l}J_{2l})] \\ &+c_{1t}^2k_t^2[k_t^2(J_{0t}^2 + J_{1t}^2) + q^2(J_{1t}^2 - J_{0t}J_{2t})] - 4c_{1l}c_{1t}\,qk_tJ_{0l}J_{1t}\} \\ &+\rho_2\{c_{2l}^2[q^2(K_{1l}^2 - K_{0l}^2) + \kappa_l^2(K_{0l}K_{2l} - K_{1l}^2)] \\ &+c_{2t}^2\kappa_t^2[\kappa_t^2(K_{1t}^2 - K_{0t}^2) + q^2(K_{0t}K_{2t} - K_{1t}^2)] - 4c_{2l}c_{2t}q\kappa_tK_{0l}K_{1t}\}\end{aligned}$$

where the notation used for $J_{m(l,t)}$ and $K_{m(l,t)}$ was introduced in section 3.2.1.

3.3. The deformation potential interaction

Substituting equations (23), (24), (15), and (16) into equation (13), and taking into account that $\nabla^2\phi = -(\omega/s_l)^2\phi$, we find

$$\hat{\mathcal{H}}_{\text{def}} = -\Xi_{\text{ac}} \sum_{mn,q} \left(\frac{\omega_{mn,q}}{as_l}\right)^2 \sqrt{\frac{\hbar}{2\pi\omega_{mn,q}\mathcal{N}\mathcal{L}}} \left[\Phi_{mn,q}\hat{b}_{mn,q} + \Phi^*_{mn,-q}\hat{b}^\dagger_{mn,-q}\right] \mathrm{e}^{\mathrm{i}m\varphi+\mathrm{i}qz/a} \tag{31}$$

where the scalar potential Φ is given by

$$\Phi_{mn,q} = \begin{cases} \mathrm{i}c_{l1}J_m(k_lr/a) & r < a \\ \mathrm{i}c_{l2}K_m(\kappa_lr/a) & r > a. \end{cases} \tag{32}$$

As expected, only the longitudinal component of the vibration contributes to a deformation potential interaction.

4. Summary

In this paper we obtained the quantization rules for acoustic vibrations confined in one, two, or all spatial dimensions and presented a general form of the deformation potential Hamiltonian in a second-quantization representation. It should be noted that the quantization rules do not change the dispersion relations and displacement pattern of the classical acoustic waves in the waveguides. However, the procedure of second quantization performed specifies uniquely the normalization constants for phonon fields and provides the operator representation for phonon variables. The formalism makes possible the consideration of processes with numbers of phonons of the order of one, and should be applied for analysis of electron–phonon interaction in mesoscopic devices.

As a specific example we considered the quantization of acoustic phonon modes in a buried cylindrical waveguide. The expressions obtained may be used for analysis of electron and phonon dynamics in buried quantum wires.

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Appendix

All three scalar potentials in equation (15) for the inner region, $r < a$, are written in terms of Bessel's functions J_m, which describe *real* vibrations involving the whole cross-section of the waveguide, provided that $k_{l,t}^2 > 0$ (i.e. $\omega < s_{(l,t)1} q_z$). In the opposite case, where $\omega > s_{l,t} q_z$, the functions J_m should be replaced by the modified Bessel functions I_m according to the identity

$$J_m(-\mathrm{i}|z|) = \mathrm{i}^m I_m(|z|). \tag{A1}$$

This situation corresponds to (interface) evanescent vibrations, exponentially decreasing toward the centre of the waveguide.

As for the outer region, $r > a$, it is characterized by the evanescent solutions (16) in terms of MacDonald's functions K_m. Vibrations are confined in the vicinity of the waveguide, provided that $\omega < s_{t2} q_z < s_{l2} q_z$. The opposite case of $\omega > s_t q_z$ can be treated formally through the substitution

$$K_m(-\mathrm{i}|z|) = \frac{\pi}{2}\,\mathrm{i}^{m+1} H_m^{(1)}(|z|). \tag{A2}$$

Here the Hankel function describes the radiation of acoustic energy from the system, which is characterized by a complex frequency spectrum.

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